

# Leakage-Resilient Public-Key Cryptography

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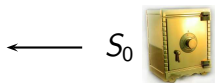
Eurocrypt 2009 Rump Session

# Black-Box Crypto

- Secret initial state  $S_0$
- On query  $X_i$ 
  - Compute  $(Y_i, S_i) \leftarrow f(X_i, S_{i-1})$ .
  - Output  $Y_i$ .

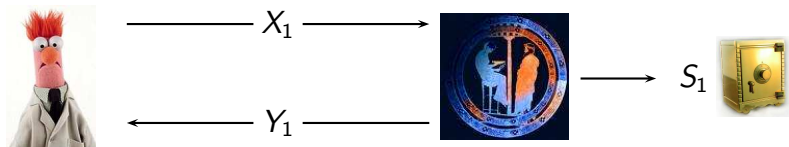


—————  $X_1$  —————>



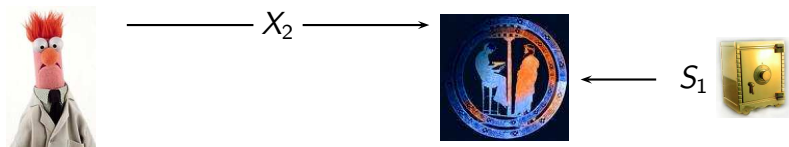
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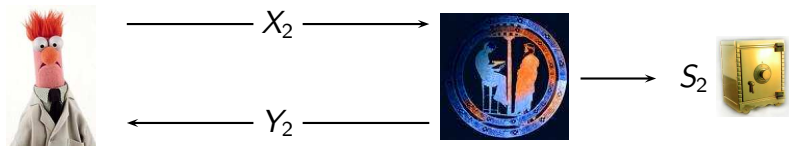
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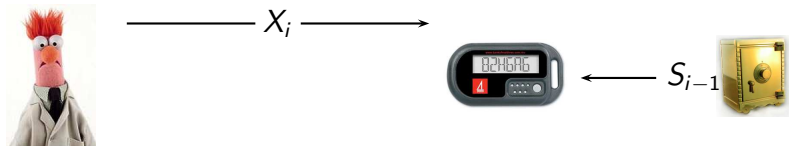


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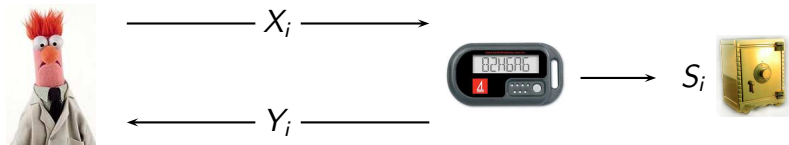
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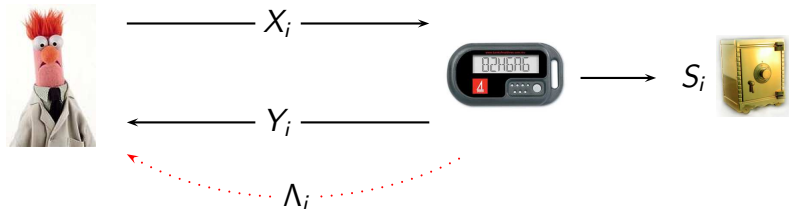
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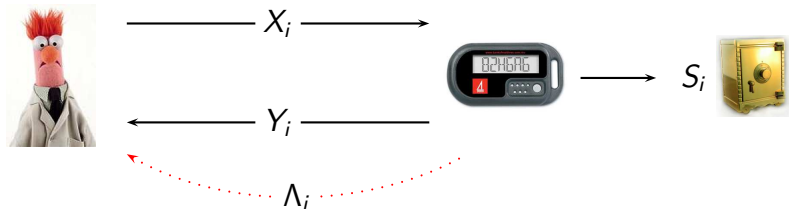
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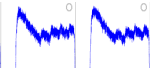




# Real-World Crypto



- Computation leaks information  $\Lambda_1, \Lambda_2, \dots$  on each invocation.
- Must prove security against **side-channel attacks**, but which?

# Some Side-Channels

-  electromagnetic radiation [QuisquaterS01]
-  power consumption [KJJ99]
-  running-time [Kocher96]
-  sound [ShamirTromer]  
`people.csail.mit.edu/tromer/acoustic`
- ...

# Leakage-Resilience

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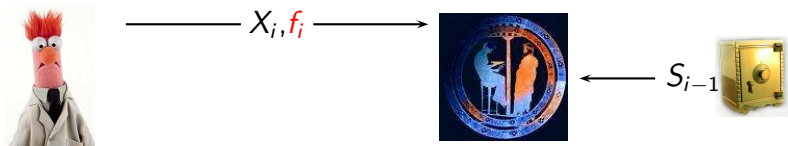
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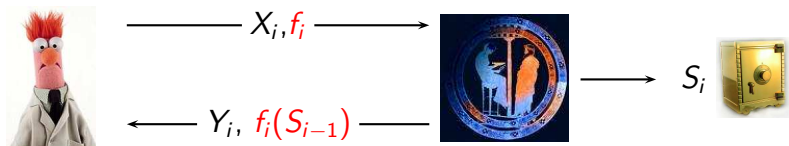
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- Need security against **all** side-channel attacks under reasonable assumption on the underlying hardware.
- Leakage Resilience [DP07]: Security against all side-channel attacks where
  - The *amount* of information leaked is bounded.
  - Only computation leaks information [MR04].

# Modelling Leakage Resilience



- $f_i$  is any *efficient* function with range  $\{0, 1\}^\lambda$ .

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## *State of leakage-resilient PKC:*

*“Tree-based” signatures not very practical, PKE open.*

## *What we want:*

*Efficient leakage-resilient signatures/PKE.*

## *Or even better:*

*Leakage-resilient instantiation of popular schemes.*

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*Leakage-Resilient instantiations of [with Eike Kiltz]*

- *PKE: Bilinear ElGamal (CCA1, CCA2?).*
- *Signatures: Waters Signatures.*

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*security proof in the **generic group model**.*

- *PKE: ElGamal.*

*under **falsifiable assumption**.*

# Bilinear ElGamal

- Cyclic groups  $\mathbb{G}, \mathbb{G}_T$  of order  $p$
- Bilinear map  $e(g^x, g^y) = e(g, g)^{xy}$ .
- Key Generation:  $x \leftarrow \mathbb{Z}_p$      $sk = g^x$      $pk = e(g, g)^x$
- Key Encapsulation:  $C \leftarrow g^r$      $K \leftarrow e(g, g)^{xr}$ .
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## Making Decapsulation Leakage Resilient

- Share  $sk = g^x$  as:  $\phi_0 = g^s$  and  $\phi_1 = g^{x-s}$
- Leakage Resilient Key Decapsulation:
  - 1  $r \leftarrow^* \mathbb{Z}_p$
  - 2  $K' \leftarrow e(C, \phi_0)$       $\phi_0 \leftarrow \phi_0 \cdot g^r$
  - 3  $K'' \leftarrow e(C, \phi_1)$       $\phi_1 \leftarrow \phi_1 \cdot g^{-r}$
  - 4  $K \leftarrow K' \cdot K''$

# “Standard” ElGamal Encryption

## Assumption

- $g$  generator of cyclic group of order  $p$ . Sample random  $x \leftarrow \mathbb{Z}_p$ ,  $\mathcal{A}$  gets  $g^x$ .
- Let  $x_1, x_2, \dots$  be random and  $x'_i \leftarrow x_i/x$  ( $x = x_i \cdot x'_i \pmod p$ )
- For  $i = 1, 2, \dots$ ,  $\mathcal{A}$  chooses  $f_i, g_i : \mathbb{Z}_p \rightarrow \{0, 1\}^\lambda$  and gets

$$f_i(x_i) \quad \text{and} \quad g_i(x'_i)$$

- $\mathcal{A}$  gets DDH challenge, i.e. must distinguish

$$g^x, g^r, g^{x \cdot r} \quad \text{from} \quad g^x, g^r, g^s$$

Any idea as to whether this problem is/isn't hard? (Easy if  $\lambda > \log p/2$ )